

EINSTEIN'S ITALIAN MATHEMATICIANS

RICCI, LEVI-CIVITA, AND THE BIRTH OF GENERAL RELATIVITY

Judith R. Goodstein



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For David, Marcia, and Mark

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Preface

This tale of the Italian mathematicians who provided Albert Einstein with the crucial mathematical architecture for general relativity might never have come about had it not been for a piece of mail that surfaced more than forty years ago in the correspondence of a very different mathematician, the Hungarian-born polymath Theodore von Kármán. He spent many years on the faculty of the California Institute of Technology, and early in my career as Caltech's first archivist, while rummaging through a collection of his letters, I came across one that he had received in 1922 from Tullio Levi-Civita, a mathematics professor in Rome. The name was unfamiliar to me, but even before reading the letter, I was struck by the handwriting. Unlike many scientists whose scrawls I had alternately puzzled over and despaired of, Levi-Civita had mastered the art of penmanship. His script was distinguished by bold, slightly exaggerated letters, with every I dotted and T crossed. His prose marched in straight, confident lines across the page, all correctly punctuated. What he had to say was at least as interesting as his handwriting: In the bitter aftermath of World War I, the victorious Allied nations, including Italy, had actively discouraged their scientists from attending meetings with their counterparts in the defeated nations of Germany and Austria-Hungary. Von Kármán, then still teaching in Germany, had evidently written to Levi-Civita to ask for his help in lifting the embargo. In his response, Levi-Civita had offered his enthusiastic support, now that “the brutal parenthesis of the war” had ended, for an international conference von Kármán proposed holding in Innsbruck, Austria. “Almost” all of his Italian colleagues would attend, he assured him, “without deplorable lingering of a bellicose mentality” still sometimes displayed by their peers.

Intrigued by the tone of the letter, I decided to investigate further. I quickly discovered that Levi-Civita had been perhaps the most brilliant and versatile member of a circle of influential Italian mathematicians—a number, like him, from Jewish backgrounds—that had flourished in the decades after the unification of Italy in 1870, only to be undone by the collapse of the nation's democratic institutions following the rise of Mussolini and fascism in the years leading up to World War II. He himself had died in 1941, stripped of his academic posts and prerogatives under the state's anti-Semitic laws and ostracized by colleagues who only a few years earlier had hailed him as their field's shining light.

My language skills did not extend to Hungarian and my German wasn't much better, but I could read and, after a fashion, speak Italian. Moreover, I would be accompanying my physicist husband, David, on his research trip to Rome in the near future. Putting aside for the moment any further inquiries into von Kármán, I found myself not long afterward standing in front of Via Sardegna 50, Levi-Civita's last known address in Rome. There I encountered a kindly janitor who told me that Levi-Civita's widow, Libera, had recently moved to another part of the city. While I stood in the empty apartment, he called her and handed me the phone. In my halting Italian, I introduced myself and explained my interest, and she invited me to visit her, which I did about a week later.

When I met Libera for the first time that summer, I knew painfully little about her and not that much more about her distinguished husband. Considering that she knew absolutely nothing about me, her subsequent generosity was astonishing. We talked for a while and then she directed me to an armoire located in an alcove adjacent to her living room, and said, "His letters are here." I already knew Levi-Civita to be a tireless correspondent, whose letters, written between 1896 and 1941, had been promised to Italy's Lincei Academy of Sciences, but I had not expected to find them tucked away in a corner of her home, numbering in the thousands.

A few weeks later we met again, and this time she introduced me to her son in law, Pier Vittorio Ceccherini, a professor of mathematics in Rome who had been a student of Beniamino Segre, now the president of the Lincei. The three of us came to an understanding that I could examine some of the letters while I was in Rome and that the Lincei would mail the complete collection to me in Pasadena, California, the following summer. They arrived still tucked in their envelopes, bundled together by year, each packet neatly tied with string. Over the next several months, I pored over their contents, consulted other sources, and began assembling a picture of Levi-Civita's life, his research, and the intellectual milieu in which he lived and worked. I discovered that he and his mathematical peers had assumed the primary role in disseminating relativity theory throughout Italy's scientific community, and that Levi-Civita himself had carried on a lively correspondence with Einstein in the years immediately before and after World War I. I presented the early fruits of this research in an invited talk at the Lincei Academy in 1973 and over the next decade published a handful of additional articles about Italy's nineteenth-century renaissance in mathematics.

Years later, having completed a book about the history of Caltech, I returned to the themes of those earlier pieces and expanded them into a book *The Volterra Chronicles*, which was published in 2007. It was sometime afterward that Sergei Gelfand, my shrewd and knowledgeable publisher at the American Mathematical Society, said to me, "What about Ricci?" Well, what about Ricci? He was certainly not one of the flashier figures of that era. Thinking back to Levi-Civita's letters, I vaguely recalled that Gregorio Ricci had been his professor and later his colleague at the University of Padua.

Their subsequent relationship had been close enough, I soon discovered, for Levi-Civita to have delivered an eloquent and emotional tribute to him at the Lincei in 1926, a year after his death. I located a copy of that talk (which is translated into English for the first time here in its entirety), read its account of the indispensable role Ricci's absolute differential calculus had played in the formulation of general relativity, and embarked on a journey that has brought me here.

The premise of this book is simply this: that the part Ricci and Levi-Civita played in contributing to Einstein's theory is a story that deserves to be told and told to a wider audience than it has reached to date. Recognizing Gregorio Ricci for his role in this saga—a mathematician who spent his whole adult life engaging in what the mathematical physicist Freeman Dyson has called “unfashionable science,” but who also had the extraordinary good fortune to encounter Tullio Levi-Civita as an effervescent student at Padua and form an enduring friendship with him—has been the driving force behind this narrative. Einstein, of course, features in this drama too—not the central figure but still a major player whose sudden, somewhat disheveled, appearance in the third act throws into sharp relief the role of its principals in setting the stage for one of the great, transformative discoveries of twentieth-century science.

Acknowledgments. This book owes much to Pier Vittorio Ceccherini and Susanna Silberstein Ceccherini, who first opened their home to me more than forty years ago and permitted me unfettered access to Levi-Civita's correspondence and private documents and other manuscript material in their possession. Tullio Ceccherini-Silberstein, their son, also a mathematician, shared family photographs with me and read the chapters as did my colleagues Tilman Sauer and Michele Vallisneri, who rescued me from time to time with their mini-tutorials on relativity and tensors. Donald Babbitt, who has been my mentor in all things mathematical, has been a loyal friend and staunch supporter for the past decade.

The publisher of the American Mathematical Society, Sergei Gelfand, made me an offer I couldn't refuse in suggesting that Ricci deserved to be brought out of the shadows. He challenged me to answer the question: “Who was he?” In 2004, Fabio Toscano published an Italian-language book about Ricci and Levi-Civita, which I found helpful.

While the considerable archival research undertaken in the preparation of this book is amply reflected in the footnotes, the kindness and generosity of the many archivists and librarians who facilitated my work deserve a special thank you. In particular, I would like to thank Orith Or Burla, Chaya Becker, and Roni Grosz, all of the Albert Einstein Archives at The Hebrew University of Jerusalem, for their help and permission to quote from The Collected Papers of Albert Einstein. In the early stages of writing this book, I was a visiting associate at the Einstein Papers Project at Caltech, courtesy

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In Padua, I had a lively correspondence with Mariarosa Davi, a teacher at the prestigious Liceo Ginnasio Tito Livio, who went beyond the call of duty to track down and send me Tullio Levi-Civita's pre-university academic records. Across the Alps, Bärbel Mund in Göttingen provided help in locating Ricci correspondence in the Felix Klein Nachlass, and Heike Hartmann offered assistance in selecting pictures in the ETH's photographic archive.

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Translations from one language to another can be unusually challenging at times. I enlisted the help of my husband, David, and our daughter, Marcia Goodstein, in translating several difficult documents from the Italian; Donald Babbitt did the same for French material. James T. Smith provided a valuable preliminary translation of Levi-Civita's memoir about Ricci's life and work.

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I owe a special debt of gratitude to Heidi Aspaturian, my long-time editor and colleague at Caltech, who through many detailed and discerning readings of the chapters helped turn the manuscript into a compelling story. As the saying goes, behind every writer is a better editor and in my case, certainly, that statement has never been truer or more appreciated.

Judith Goodstein
July 2017

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Photo 1. Ricci Curbastro family tree.

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Photo 30. Gregorio Ricci Curbastro and family, Sant’ Agata sul Santerno (ca. 1921).

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Photo 2. Gregorio Ricci Curbastro (ca. 1869). Credit: Ida Ricci Curbastro/Biblioteca Comunale Fabrizio Trisi.

Photo 3. Postcard scene, Lugo (ca. 1900).

Photo 10. Bianca Bianchi Ricci Curbastro. Credit: Maria Giovanna Ricci Curbastro/Biblioteca Comunale Fabrizio Trisi, Lugo.

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Photo 22. Title page of the “Outline [or preliminary version] of a Generalized Theory of Relativity and of a Theory of Gravitation.” Credit: Paul Epstein Papers, Courtesy of the Archives, California Institute of Technology.

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Photo 31. Postcard from Levi-Civita to Theodore von Kármán, 1921. Credit: Courtesy of the Archives, California Institute of Technology.

Ceccherini-Silberstein Family

Photo 14. Signed portrait of Garibaldi.

Photo 23. Tullio Levi-Civita (1873–1941) (ca. 1912).

Photo 28. Passport for Libera Levi-Civita, issued in Padua, Sept. 30, 1918.

Photo 29. Gregorio Ricci Curbastro (1852–1925).

Photo 32. Tullio Levi-Civita, La Plata, Argentina, 1923.

Enrico Persico Papers, Dept. of Physics, Sapienza University, Rome

Photo 33. A humorous depiction of Levi-Civita revealing the mysteries of the absolute differential calculus (ca. 1923).

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Fondo ‘Ricci Curbastro,’ Il Liceo Statale ‘Ricci Curbastro’ di Lugo (RA), Italy, III Raccoglitore

Photo 8. Pages from the manuscript for Ricci’s inaugural lecture in Padua, 1881. Credit: Fondo ‘Ricci Curbastro,’ Il Liceo Statale ‘Ricci Curbastro’ di Lugo (RA) Italy, III Raccoglitore, #48.

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Photo 4. Palazzo della Sapienza.

Photo 27. Caffè Pedrocchi, Padua (ca. 1915).

Photo 34. Tullio Levi-Civita and his wife Libera Trevisani.

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Photo 25. Pages from a Levi-Civita letter to Einstein, March 28, 1915. © The Hebrew University of Jerusalem, Israel.

Photo 26. Two postcards from Einstein to Levi-Civita, April 14 and April 21, 1915. © The Hebrew University of Jerusalem.

Ida Ricci Curbastro

Photo 2. Gregorio Ricci Curbastro (ca. 1869). Credit: Ida Ricci Curbastro/Biblioteca Comunale Fabrizio Trisi.

Institut Mittag-Leffler

Photo 6. Ulisse Dini (1845-1918).

Maria Giovanna Ricci Curbastro

Photo 10. Bianca Bianchi Ricci Curbastro. Credit: Maria Giovanna Ricci Curbastro/Biblioteca Comunale Fabrizio Trisi, Lugo.

Photo 11. Bianca Bianchi Ricci Curbastro.

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Appendix C. “Tullio Levi-Civita” by W. V. D. Hodge is reprinted with permission of the publisher, The Royal Society, from *Obituary Notices of Fellows of the Royal Society of London*, 4 (1942), 151–165.

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Senato della Repubblica

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Yale University

Photo 18. Ricci-Levi-Civita’s paper published in *Mathematische Annalen* 54 (October, 1900). Credit: Cushing/Whitney Medical Library, Yale University. Public Domain.

Selected References

- [1] Accademia Nazionale dei Lincei. Tullio Levi-Civita. *Convegno internazionale celebrativo del centenario della nascita: Roma, 17–19 dicembre 1973*. Rome, 1975.
- [2] U. Amaldi, *Commemorazione del socio Tullio Levi-Civita* (Italian), *Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat.* (8) **1** (1946), 1130–1155. MR0020061
- [3] Max Born and Max von Laue, “Max Abraham.” *Physikalische Zeitschrift*, 24 (1923): 49–53.
- [4] R. J. B. Bosworth, *Mussolini’s Italy: Life under the Fascist Dictatorship, 1915–1945*. New York, 2007.
- [5] Umberto Bottazzini, *Ricci and Levi-Civita: from differential invariants to general relativity*, *The symbolic universe* (Milton Keynes, 1996), Oxford Univ. Press, New York, 1999, pp. 241–259. MR1750739
- [6] Carl B. Boyer, *A history of mathematics*, John Wiley & Sons, Inc., New York-London-Sydney, 1968. MR0234791
- [7] Carlo Cattani and Michelangelo De Maria, *Max Abraham and the reception of relativity in Italy: his 1912 and 1914 controversies with Einstein*, *Einstein and the history of general relativity* (North Andover, MA, 1986), *Einstein Stud.*, vol. 1, Birkhäuser Boston, Boston, MA, 1989, pp. 160–174. MR1200721
- [8] ———, “The 1915 Epistolary Controversy between Einstein and Tullio Levi-Civita.” In Don Howard and John Stachel, eds., *Einstein and the History of General Relativity*, 175–200. Boston, 1989.
- [9] Emanuele Cenghiaro and Pier Giovanni Zanetti. *Padova e la Grande Guerra*. Padua, 2008.
- [10] Claudio Citrini. “Matematica e vita civile nel Politecnico di cento anni fa: la vicende di Max Abraham.” *Annali di Storia delle Università Italiane* 12 (2008): 101–117.
- [11] Umberto Cisotti and Carlo Somigliana. “Tullio Levi-Civita.” *R. Istituto Lombardo di Scienze, lettere Arti. Rendiconti*, 75 (1942): 110–112.
- [12] Salvatore Coen (ed.), *Mathematicians in Bologna 1861–1960*, Birkhäuser/Springer Basel AG, Basel, 2012. MR2934582
- [13] *The Collected Papers of Albert Einstein*, 14 vols. Princeton, 1987–2015.
- [14] Freeman J. Dyson, *Unfashionable pursuits*, *Math. Intelligencer* **5** (1983), no. 3, 47–54. MR737690
- [15] Albert Einstein, *Autobiographical notes*, A bilingual German/English edition, Open Court Publishing Co., La Salle, Ill., 1979. Translated from the German and edited by Paul Arthur Schilpp. MR529428
- [16] ———, *Relativity: The Special and General Theory*. transl. R. W. Lawson, New York, 1921.
- [17] Luther Pfahler Eisenhart, *Riemannian Geometry*, Princeton University Press, Princeton, N. J., 1949. 2d printing. MR0035081
- [18] ———, “Ricci’s Principal Directions for a Riemann Space and the Einstein Theory.” *Proceedings of the National Academy of Sciences* 8 (1922): 24–26.

- [19] Jean Eisenstaedt, *The curious history of relativity*, Princeton University Press, Princeton, NJ, 2006. How Einstein's theory of gravity was lost and found again; Translated from the 2003 French original by Arturo Sangalli; With a foreword by Thibault Damour. MR2264671
- [20] Philipp Frank, *Einstein: His Life and Times*, transl. G. Rosen, New York, 1947.
- [21] Peter Louis Galison, "Minkowski's Space-Time: From Visual Thinking to the Absolute World." *Historical Studies in the Physical Sciences* (1979): 85–121.
- [22] Livia Giacardi and Rossana Tazzioli, eds., *Le lettere di Eugenio Beltrami a Betti, Tardy and Gherardi*. Milan, 2012.
- [23] Stanley Goldberg, *Understanding relativity*, Birkhäuser Boston, Inc., Boston, MA, 1984. Origin and impact of a scientific revolution. MR736684
- [24] Judith R. Goodstein, "Levi-Civita, Einstein, and Relativity in Italy." *Atti Rendiconti Accademia dei Lincei* 8 (1975): 43–51.
- [25] ———, "The Italian Mathematicians of Relativity." *Centaurus* 26 (1983): 241–261.
- [26] ———, *The Volterra Chronicles: The Life and Times of an Extraordinary Mathematician, 1860–1940*. AMS, Providence, 2007.
- [27] *Gregorio Ricci Curbastro: La vita di un liceo e l'opera di un matematico*, C. Bartolotti and A. Pirazzini, eds., Lugo, 2002.
- [28] Hanoch Gutfreund, and Jürgen Renn. *The Road to Relativity: The History and Meaning of Einstein's "The Foundation of General Relativity."* Princeton, 2015.
- [29] Walter Isaacson. *Einstein: His Life and Universe*. New York, 2007.
- [30] Michel Janssen and Jürgen Renn, eds., "Arch and Scaffold: How Einstein Found his Field Equations." *Physics Today* 68 (2015): 30–36.
- [31] Louis Kollros. "Albert Einstein en Suisse. Souvenirs." *Helvetica Physica Acta Supplement* 4 (1956): 271–281.
- [32] Tullio Levi-Civita, *Opere matematiche. Memorie e note. Vol. I. 1893–1900* (Italian), Nicola Zanichelli Editore, Bologna, 1954. Pubblicate a cura dell'Accademia Nazionale dei Lincei. MR0062680
- [33] ———, *Lezioni di calcolo differenziale assoluto*, ed. Enrico Persico, Rome, 1925; *The Absolute Differential Calculus (calculus of tensors)*, ed. E. Persico, Transl. M. Long, London, 1927.
- [34] ———, "Commemorazione del Socio Nazionale Gregorio Ricci-Curbastro." *Mem. Accad. Lincei* 1 (1926): 555–567.
- [35] Sandra Linguerrì and Raffaella Simili, eds. *Einstein parla italiano: Itinerari e polemiche*. Bologna, 2008.
- [36] MacTutor History of Mathematics archive. Available at <http://www-history.mcs.st-andrews.ac.uk/index.html>.
- [37] Alberto Malfitano, Gian D. Veggi, and Antonio Curzi. *1859: la Romagna e Lugo dalle Legazioni al Regno d'Italia*. Lugo di Romagna, 2010.
- [38] Pietro Nastasi and Rosanna Tazzioli. *Aspetti scientifici e umani nella corrispondenza di Tullio Levi-Civita (1873–1941)*. Palermo, 2000.
- [39] ———, "Toward a Scientific and Personal Biography of Tullio Levi-Civita (1873–1941)." *Historia Mathematica* 32 (2005): 203–236.
- [40] John Norton, *How Einstein found his field equations, 1912–1915*, Einstein and the history of general relativity (North Andover, MA, 1986), Einstein Stud., vol. 1, Birkhäuser Boston, Boston, MA, 1989, pp. 101–159. MR1200720
- [41] ———, "A Peek into Einstein's Zurich Notebook." Available at <http://www.pitt.edu/~jdnorton/Goodies>.
- [42] Abraham Pais, *Subtle is the Lord...*, The Clarendon Press, Oxford University Press, New York, 1982. The science and life of Albert Einstein. MR690419
- [43] Karin Reich, *The American contribution to the theory of differential invariants, 1900–1916*, The attraction of gravitation: new studies in the history of general relativity

- (Johnstown, PA, 1991), *Einstein Stud.*, vol. 5, Birkhäuser Boston, Boston, MA, 1993, pp. 225–247. MR1735383
- [44] Gregorio Ricci-Curbastro, *Opere. Note e Memorie*, 2 vols. Rome, 1956.
- [45] ———, “Direzioni e invarianti principali in una varietà qualunque.” *Atti del Reale Istituto Veneto di Scienze, Lettere ed Arti* 63 (1904): 1233–1239.
- [46] M. M. G. Ricci and T. Levi-Civita, *Méthodes de calcul différentiel absolu et leurs applications* (French), *Math. Ann.* **54** (1900), no. 1-2, 125–201. MR1511109
- [47] Jürgen Renn, “The Summit Almost Scaled: Max Abraham as a Pioneer of a Relativistic Theory of Gravitation.” In J. Renn, ed., *The Genesis of General Relativity*, vol. 3, 1229–1255. Dordrecht, 2007.
- [48] David E. Rowe, *Einstein’s gravitational field equations and the Bianchi identities*, *Math. Intelligencer* **24** (2002), no. 4, 57–66. MR1930868
- [49] H. S. Ruse, *Obituary: Tullio Levi-Civita*, *Edinburgh Math. Notes* **33** (1943), 19–24. MR0008797
- [50] Tilman Sauer, “Albert Einstein, Review Paper on General Relativity Theory (1916).” In I. Grattan-Guinness, ed., *Landmark Writings in Western Mathematics, 1640–1940*, 802–822. Amsterdam, 2005.
- [51] ———, “Marcel Grossmann and his contribution to the general theory of relativity.” [arXiv:1312.4068v2 [physics.hist-ph]]
- [52] Carl Seelig, *Albert Einstein: A Documentary Biography*, transl. M. Savill, London, 1956.
- [53] Francesco Severi, “Discorso.” In *Celebrazione in Lugo del centenario della nascita di Gregorio Ricci Curbastro*, 55–70. Lugo, 1954.
- [54] Carlo Somigliana, *Obituary: Tullio Levi-Civita e Vito Volterra* (Italian), *Rend. Sem. Mat. Fis. Milano* **17** (1946), 1–15. MR0024376
- [55] Dirk J. Struik, “Outline of a History of Differential Geometry.” *Isis* 20 (1933): 161–191.
- [56] ———, *A Concise History of Mathematics*. New York, 1967.
- [57] ———, “J. A. Schouten and the Tensor Calculus.” *Nieuw Archief Voor Wiskunde* 26 (1978): 96–107.
- [58] Dirk J. Struik, *Schouten, Levi-Civita, and the emergence of tensor calculus*, The history of modern mathematics, Vol. II (Poughkeepsie, NY, 1989), Academic Press, Boston, MA, 1989, pp. 99–105. MR1037810
- [59] ———, “From Riemann to Ricci: The Origins of the Tensor Calculus.” In H. M. Srivastava and Th. M. Rassias, eds., *Analysis, Geometry and Groups: A Riemann Legacy Volume*, 657–674. Palm Harbor, 1993.
- [60] Tina Tomasi and Nella Sistoli Paoli, *La Scuola normale di Pisa dal 1813 al 1945: cronache di un’istituzione*. Pisa, 1990.
- [61] Fabio Toscano, *Il genio e il gentiluomo: Einstein e il matematico italiano che salvò la teoria della relatività generale*. Milan, 2004.
- [62] ———, “Luigi Bianchi, Gregorio Ricci Curbastro e la scoperta delle identità di Bianchi.” In *Atti del XX Congresso Nazionale di Storia della Fisica e dell’astronomia*, ed. E. Schettino, 353–370. Naples, 2001.
- [63] Angelo Ventura, *Padova*. Bari, 1989.

Notes

Abbreviations

ACS	Archivio Centrale dello Stato (Rome)
ASR	Archivio di Stato, Roma
ASUR	Archivio Storico dell'Università di Roma
CPAE	The Collected Papers of Albert Einstein
MI	Ministero dell'Interno
MPI	Ministero della Pubblica Istruzione–Direzione Generale Istruzione Superiore
SUB Göttingen	Niedersächsische Staats- und Universitätsbibliothek Göttingen
UIS	Università e Istruzione Superiore
DGIU	Direzione Generale Istruzione Universitaria

Notes for Chapter 1

1. Gregorio Ricci Curbastro [surname hereafter given as Ricci], *Suor Vincenza Ricci Curbastro, Figlia della Carità* (Padua, 1896), 6.
2. Quoted in Fabio Toscano, *Il genio e il gentiluomo: Einstein e il matematico italiano che salvò la teoria della relatività generale* (Milan, 2004), 129.
3. G. Ricci, *Suor Vincenza*, 6.
4. Francesco Taglioni, “Attestato di studio,” October 15, 1869, ASR, Fondo Università, busta 590, fasc. 7700.
5. In 1935, the university moved to its present site near the train station.
6. Denis Mack Smith, *Modern Italy: A Political History*, rev. ed. (Ann Arbor, 1969), 86.
7. Charles Dickens, *Pictures from Italy*, edited with notes and an introduction by Kate Flint (London, 1998), 150.
8. Gaspare Finali to Cesare Correnti, October 24, 1871, ACS, MPI, Personale, busta 1788, fasc. Gregorio Ricci.
9. Dora Dumont, “The Nation As Seen from Below: Rome in 1870,” *European Rev. of History* 15 (2008), 489.
10. G. Ricci to Angelo Manzoni, November 24, 1872, quoted in F. Toscano, *Il genio*, 134.

Notes for Chapter 2

1. Quoted in Giovanni Sansone, *Algebristi, analisti, geometri differenzialisti, meccanici e fisici-matematici ex-normalisti del periodo 1860-1929* (Pisa, 1977), 12.
2. Vito Volterra, Senato del Regno, Atti parlamentari. Discussioni. November 22, 1918.
3. G. Ricci to Alfredo Capelli, March 17, 1895, Fondo Ricci Curbastro, Biblioteca Trisi, Lugo.
4. Ibid.
5. G. Ricci, "Cenno sulla vita scientifica del Dott. Gregorio," February 18, 1880, ASR, Fondo Università, busta 1089 (miscellanea), fasc. documenti e pubblicazione relativi all'assistente universitaria Gregorio Ricci.
6. An undated document listed as #52 in the inventory of the Ricci papers preserved in the Liceo Scientifico di Lugo.
7. Enrico Betti, "Attestato," March 20, 1877, ASR, Fondo Università, busta 590, fasc. 7700.
8. "Sopra un sistema di due equazioni differenziali lineari di cui l'una è quella dei fattori integranti dell'altra" (1877).
9. "Sulla teoria elettrodinamica di Maxwell" (1877); "Sopra la deduzione di una nuova legge fondamentale di elettrodinamica" (1877).
10. An undated document, item #52, Ricci papers, Liceo Scientifico di Lugo.

Notes for Chapter 3

1. E. Betti to unnamed official, July 18, 1876, ACS, MPI, Personale, busta 1788.
2. Ibid.
3. G. Ricci, *Suor Vincenza*, 8.
4. Domenico Ricci to G. Ricci, undated, Fondo Ricci Curbastro, Biblioteca Fabrizio Trisi, Lugo.
5. J. Plücker, *Neue Geometrie des Raumes, gegründet auf die Betrachtung der geraden Linie als Raumelement, mit einem Vorwort von A. Clebsch* (Leipzig, 1868-69).
6. David E. Rowe, "Klein, Hilbert, and the Göttingen Mathematical Tradition," *Osiris* 5 (1989), 192.
7. Felix Klein, "Vergleichende Betrachtungen über neuere geometrische Forschungen," vol. 1 of *Gesammelte mathematische Abhandlungen* (Berlin, 1921), 460-497.
8. F. Klein, *Development of Mathematics in the 19th Century*, trans. M. Ackerman (Brookline, 1979), 155.

9. The official name of the institution was the *Königlich Bayerische Technische Hochschule München*; in 1901, it gained the right to award doctorates and in 1970 changed its name to the Technische Universität München.
10. Quoted in Livia Giacardi, “Models in Mathematical Teaching in Italy (1850-1950),” <http://math-art.eu/Cagliari2013-Lectures.php>. See also, D. Rowe, “Mathematical models as artefacts for research: Felix Klein and the case of Kummer surfaces,” *Mathematische Semesterberichte*, 60 (2013), 1–24.
11. The 1878-1879 academic year in Munich consisted of only two semesters: a winter semester that ended in March, and a summer semester that began after Easter and ended in July.
12. G. Ricci to E. Betti, December 29, 1878, Betti Archive, Archivio della Scuola Normale Superiore di Pisa.
13. Ibid.
14. G. Ricci, “Relazione di Gregorio Ricci Curbastro sul perfezionamento svolto a Monaco di Baviera presso il prof. Klein,” Mar. 14, 1879, ACS, MPI, Personale, busta 1788.
15. F. Klein, “Lettera del prof. Klein,” March 13, 1879, ACS, MPI, Personale, b. 1788.
16. F. Klein, *Development of Mathematics*, 348. His papers on the subject were published in 1878 and 1879 in *Math. Ann.*
17. G. Ricci, “Relazione semestrale di G. Ricci,” ACS, MPI, Personale, busta 1788.
18. F. Klein, “Lettera del prof. Klein,” July 29, 1879, *ibid.*
19. F. Klein, *Development of Mathematics*, 278.
20. G. C. Young, “Professor Klein,” *The Times*, 1925.
21. G. Ricci to F. Klein, November 3, 1892, SUB Göttingen, Cod. Ms. F. Klein 11:500.

Notes for Chapter 4

1. Ulisse Dini to F. Mazzuoli, November 12, 1879, ACS, MPI, Personale, busta 1788.
2. U. Dini to unknown correspondent, Nov. 13, 1879, *ibid.*
3. G. Ricci to E. Betti, December 3, 1880, Betti Archive, Scuola Normale Superiore di Pisa.
4. E. Betti to G. Ricci, undated draft, *ibid.*
5. V. Volterra, “Betti, Brioschi, Casorati: Tre analisti e tre modi di considerare le questioni d’analisi,” *Saggi scientifici* (Bologna, 1920; reprint, with an introduction by Raffaella Simili, Bologna, 1990), 49.
6. G. Ricci, “Origini e sviluppo dei moderni concetti fondamentali sulla geometria,” *Opere*, vol. 2 (Rome, 1956-57), 292.
7. Quoted in Tullio Levi-Civita, “Gregorio Ricci Curbastro,” *Opere*, vol. 1, 1.

8. F. Klein, *Development of Mathematics*, 203.
9. T. Levi-Civita, “Commemorazione del socio nazionale prof. Gregorio Ricci Curbastro,” *Mem. Acc. Lincei* 1(1926), 557.
10. D. Ricci to G. Ricci, January 20, 1881, Fondo Ricci Curbastro, Biblioteca, Lugo.
11. T. Levi-Civita, “Commemorazione,” 561.
12. Angelo Tonolo, “Commemorazione di Gregorio Ricci Curbastro nel primo centenario della nascita,” *Rendiconti del Seminario della Università di Padova*, 23 (1954), 19.
13. Ibid.
14. Maria Pastori, “Necrologio di Angelo Tonolo,” *Bollettino dell’Unione Matematica Italiana*, 17 (1962), 423.
15. Livia Giacardi, “Federigo Enriques (1871-1946) and the training of mathematics teachers in Italy,” in Salvatore Coen, ed., *Mathematicians in Bologna 1861-1960* (Basel, 2012), 211.
16. Giuseppe Bettamini *et al* to “Onoverole Sig. Rettore,” December 11, 1882, ACS, MPI, Personale, busta 1788.
17. Terenzio Mamiani to Guido Baccelli, January 27, 1883, *ibid.*
18. Francesco Brioschi to Michele Coppino, November 26, 1884, *ibid.*
19. Aldo Finzi to T. Levi-Civita, May 19, 1926, Levi-Civita Personal Papers, courtesy of Ceccherini-Silberstein family.

Notes for Chapter 5

1. Antonio Ricci to G. Ricci, July 27, 1879, Fondo Ricci Curbastro, Biblioteca di Lugo.
2. Bianca Bianchi Azzarani to G. Ricci, January 5, 1884, Fondo Ricci Curbastro, Biblioteca di Lugo.
3. *Ibid.* January 20, 1884.
4. *Ibid.* January 18, 1884.
5. *Ibid.*
6. *Ibid.* February 5, 1884.
7. Quoted by Dirk Struik, in “From Riemann to Ricci: The Origins of the Tensor Calculus,” H. M. Srivastava, T. M. Rassias, eds., *Analysis, Geometry, and Groups: A Riemann Legacy* (Palm Harbor, 1993), 657.
8. Bernhard Riemann, “Über die Hypothesen welche der Geometrie zu Grunde liegen” (“On the Hypotheses That Lie at the Foundations of Geometry”).
9. B. Riemann, “Über eine Frage der Wärmeleitung” (“On a Question of Heat Conduction”).

10. Olivier Darrigol, “The Mystery of Riemann’s Curvature,” *Historia Mathematica* 42 (2015), 48.
11. F. Klein, *Development of Mathematics*, 237.
12. G. Ricci, “Principii di una Teoria delle Forme Differenziali Quadratiche” (“Principles of a theory of differential quadratic forms”), *Opere*, vol. 1, 140.
13. B. Bianchi to G. Ricci, March 18, 1884, Fondo Ricci Curbastro, Biblioteca di Lugo.

Notes for Chapter 6

1. *La Venezia*, 24 February 1887.
2. T. Levi-Civita, “Commemorazione,” 560.
3. Francesco G. Tricomi, “Matematici italiani del primo secolo dello stato unitario,” *Mem. Acc. Sci. Torino*, Classe di Scienze Fisiche, Matematiche e Naturali, 4th ser. 1 (1962), 41.
4. Giuseppe Veronese to M. Coppino, July 8, 1884, ACS, MPI, DGIS, UIS, busta 563, fasc. 510, sottofasc. 4.
5. Luigi Cremona, August 1886, *ibid.*
6. Quoted in “Seduta straordinaria, dei soli Professori Ordinari, 9 Novembre 1886,” *ibid.* All details of this meeting, unless otherwise noted, are from this source.
7. See Giovanni Canestrini to Gianpaolo Vlacovich, October 26, 1886; G. Vlacovich to M. Coppino, October 30, 1886; and G. Veronese to Commendatore [unidentified], November 2, 1886, *ibid.*
8. U. Dini to M. Coppino, December 5, 1886, ACS, MPI, DGIS, UIS, busta 465, fasc. 23, sottofasc. 4.
9. G. Ricci to M. Coppino, December 3, 1886, *ibid.*
10. Giuseppe Lorenzoni, Giovanni Omboni, Ernesto Padova, Francesco D’Arcais, Augusto Righi, to M. Coppino, Feb. 28, 1887, ACS, MPI, UIS, busta 563, fasc. 510, sottofasc. 4.
11. G. Canestrini to G. Vlacovich, March 3, 1887, *ibid.*
12. *Ibid.*
13. G. Ricci to M. Coppino. Dec. 3, 1886, ACS, MPI, DGIS, UIS, busta 465, fasc. 23, sottofasc. 4.
14. G. Ricci to M. Coppino, June 19, 1887, ACS, MPI, DGIS, UIS, busta 563, fasc. 510, sottofasc. 4.
15. U. Dini to Giovanni Ferrando, June 18, 1887, *ibid.*
16. G. Ricci to V. Volterra, May 15, 1888, Vito Volterra Papers, Accademia Nazionale dei Lincei, Rome.

Notes for Chapter 7

1. T. Levi-Civita, “Commemorazione,” 561.
2. It came with a cash prize of 10,000 *lire* to be awarded annually for the best papers or discoveries in the physical, mathematical, and natural sciences, or the moral, historical, and philological sciences.
3. In private, Beltrami described the fourth entry, submitted by G. Ribaldi, an engineer, as trivial and full of errors, and barely mentioned it in his report.
4. Eugenio Beltrami to E. Betti, April 12, 1889, in L. Giacardi and R. Tazzioli, eds., *Le lettere di Eugenio Beltrami a Betti, Tardy e Gherardi* (Milan, 2012), 163.
5. E. Beltrami to E. Betti, May 9, 1889, *Le lettere*, 164.
6. E. Beltrami to E. Betti, June 11, 1889, *ibid.*, 165.
7. “Relazione sul concorso al premio Reale per la Matematica per l’anno 1887,” *Atti della R. Accademia dei Lincei* 5 (1889), 307.
8. E. Beltrami, “Saggio di interpretazione della geometria non-euclidean,” *Giornale di matematiche* 6 (1868), 284-312.
9. D. Struik, “From Riemann to Ricci,” 666.
10. G. Ricci, “Principii di una teoria delle forme differenziali quadratiche,” *Opere*, vol. 1, 139-140.
11. E. Beltrami, “Relazione sul concorso al premio Reale per la Matematica per l’anno 1887,” 304. For an illuminating study of how the Italian mathematical community viewed Ricci’s work, see also Umberto Bottazzini, “Ricci and Levi-Civita: From Differential Invariants to General Relativity,” in Jeremy Gray, *The Symbolic Universe: Geometry and Physics 1890-1930* (Oxford, 1999), 241-258.
12. E. Beltrami, 305.
13. G. Ricci, “Sui parametri e gli invarianti delle forme differenziali quadratiche,” *Opere*, vol. 1, 177-188.
14. Quoted in U. Bottazzini, “Ricci and Levi-Civita,” 244.
15. F. Toscano, *Il genio e il gentiluomo*, 164.
16. For a more detailed exposition, see Appendix A, page 133.
17. “Sulla derivazione covariante ad una forma quadratica differenziale,” *Opere*, vol. 1, 199-203.
18. Quoted in U. Bottazzini, “Ricci and Levi-Civita,” 245-246.
19. Quoted in T. Levi-Civita, “Commemorazione,” 558.
20. *J. Reine Angew. Math.* 70 (1869), 46-70.
21. Quoted in O. Darrigol, “The Mystery of Riemann’s Curvature,” 70.

22. T. Levi-Civita, *Lezioni di calcolo differenziale assoluto* (Rome, 1925), vii.
23. D. Struik, “From Riemann to Ricci,” 663.
24. *Ibid.*, 665.
25. Luca Dell’Aglia, “On the Genesis of the Concept of Covariant Differentiation,” *Revue d’histoire des mathématiques* 2 (1996), 255.
26. G. Ricci, “Delle derivazioni covarianti,” 1888, 245. The independence of the form of equations with respect to changes in coordinate systems (which Ricci mentioned several times) was central to Einstein’s adopting differential geometry for general relativity. In special relativity, Einstein managed to make the equations of motion (and of electromagnetism) invariant in form for the class of observers moving with constant velocity. (This solved the ether problem, because the speed of light emerged from the theory as invariant for all these observers.)
 In general relativity, Einstein sought to make equations invariant in form for any observer, including accelerated ones. This was later known as the principle of general covariance. (A more mature understanding of general covariance shows that it does not really cover a general class of observers, since these cannot really be identified with coordinate systems in general. But what general covariance does is to establish an even more important principle of general relativity—that there is no prior space-time geometry, but rather that it arises naturally from the solutions of the Einstein field equations.)
27. G. Ricci, “Delle derivazioni covarianti,” *Opere*, vol. 1, 247. This is actually standard procedure in general relativity today. It appears everywhere in the Charles Misner, Kip Thorne, and John Wheeler textbook on gravitation, for instance: choose a convenient reference frame (such as a “local rest frame”) to write equations conveniently using tensors, then elevate them automatically to general coordinates because their form does not change.
28. E. Beltrami, “Relazione,” 306–307.
29. *Ibid.*, 246.
30. *Ibid.*
31. Francesco Porro to T. Levi-Civita, May 5, 1926, Levi-Civita Personal Papers.
32. Valentino Cerruti, “Relazione sul concorso al Premio Reale per la Matematica per l’anno 1889,” *Rendiconti delle sedute solenni della R. Accademia dei Lincei del 5 giugno 1892*, vol. 8, 40.
33. G. Ricci, “Di alcune applicazioni del calcolo differenziale assoluto alla teoria delle forme differenziali quadratiche binarie e dei sistemi a due variabili,” 1893, *Opere*, vol. 1, 311.
34. T. Levi-Civita, “Commemorazione,” 557–558.

Notes for Chapter 8

1. Angelo Ventura, *Padova* (Bari, 1989), 120–121.
2. Mariarosa Davi, “I duecento anni del liceo Tito Livio,” *Padova e il suo territorio*, 27, no. 160 (2012), 25–29. For a full account of his career, see Davi, “Giacomo Levi-Civita amministratore cittadino,” M. Davi and Giulia Simone, eds., *Giacomo Levi-Civita e l’ebraismo veneto tra Otto e Novecento* (Padua, 2015), 15–34.
3. Information about Giacomo Levi-Civita’s early life also draws on documents in the Archivio Storico del Senato della Repubblica, Rome, and from an unpublished interview by the author with Susanna Silberstein Ceccherini, Caprino Veronese, July 18, 2009. [Hereafter referred to as S.S.C. Interview]
4. Annie Sacerdoti and Luca Fiorentino, *Guida all’Italia ebraica* (Genoa, 1986), 119. See also, Ariel Viterbo, “Dal primo dopoguerra alla Shoà,” in Claudia De Benedetti, ed., *Il cammino della Speranza. Gli ebrei e Padova* (Padua, 2000), vol. II, 133; Francesco Selmin, *Nessun “giusto per Eva.” La Shoah a Padova e nel Padovano* (Sommacampagna, 2011).
5. Università La Sapienza, Archivio Storico, Tullio Levi-Civita, AS 487, no. 03, “R. Università degli Studi di Roma. Stato matricolare,” undated.
6. For the complete list of courses, see files in the Liceo Tito Livio archives, “Registro della quinta ginnasio, 1886–1887” and “Elenco degli alunni premiati, 1886–1887,” busta Atti 1886–1887.
7. Felix J. D. Carli to T. Levi-Civita, October 12, 1930, Tullio Levi-Civita Papers, Accademia dei Lincei, Rome.
8. Ugo Amaldi, “Commemorazione del Socio Tullio Levi-Civita,” *Rendiconti dell’Accademia Nazionale dei Lincei*, ser. 7, vol. 1 (1946), 1135.
9. S.S.C. Interview.
10. E-mail from M. Davi to the author, August 2, 2014.
11. U. Amaldi, “Commemorazione,” 1135.
12. William Valance Douglas Hodge, “Tullio Levi-Civita, 1873–1941,” *Obituary Notices of Fellows of the Royal Society of London*, 4 (1942), 152.
13. W. V. D. Hodge, “Tullio Levi-Civita,” 157.
14. *Ibid.*, 152.
15. Adolfo Bartoli to Paolo Boselli, April 27, 1890, ACS, MPI, DGIS, UIS, busta 563, fasc. 797, sottofasc. 8.
16. Memorandum, A. Bartoli to P. Boselli, December 11, 1889, *ibid.*
17. G. Ricci to V. Volterra, Jan. 9, 1890, and V. Volterra to G. Ricci, Jan. 11, 1890, Vito Volterra Collection, Accademia Nazionale dei Lincei, Rome.
18. D. Ricci to G. Ricci, February 4, 1890, Fondo Ricci Curbastro, Biblioteca, Lugo.

19. U. Dini to P. Boselli, April 2, 1890, ACS, MPI, DGIS, UIS, busta 563, fasc. 797, sottofasc. 8.
20. L. Cremona, “Consiglio Superiore di Pubblica Istruzione, Estratto di verbale dell’adunanza,” October 28, 1890, ACS, MPI, DGIS, UIS, busta 948, fasc. 797, sottofasc. 8.
21. T. Levi-Civita, “Commemorazione,” 555.
22. *Ibid.*, 561.
23. His academic studies through these years can be followed in Università degli Studi di Padova, Archivio Storico, Registro della carriera scolastica Tullio Levi-Civita.
24. G. Veronese, *Fondamenti di geometria a più dimensioni e a più specie di unità rettilinee esposti in forma elementare: Lezioni per la scuola di Magistero in matematica* (Padua, 1891).
25. Detlef Laugwitz, “Debates about Infinity in Mathematics around 1890: The Cantor–Veronese Controversy, its Origins and its Outcome,” *NTM International Journal of History and Ethics of Natural Sciences, Technology & Medicine*, 10 (2002), 103; see also, D. Laugwitz, “Tullio Levi-Civita’s Work on Nonarchimedean Structures,” in *Levi-Civita: Convegno internazionale celebrativo del centenario della nascita*, 8 (Rome, 1975), 297-311.
26. T. Levi-Civita, “Sugli infiniti ed infinitesimi attuali quali elementi analitici,” *Atti Inst. R. Veneto di Sc., lett., ed arti*, 4(1892–1893), 1765–1815. The algebraic work is regarded today as a precursor to non-standard analysis.
27. U. Amaldi, “Commemorazione del Socio Tullio Levi-Civita,” 1132.
28. T. Levi-Civita, “Sui numeri transfiniti,” *Rend. Acc. dei Lincei*, 7 (1898), 91–96.
29. The following titles are listed in Registro della carriera scolastica Vol. D, no. 125, Università di Padova, Archivio Storico, Tullio Levi-Civita: “I numeri reali ammettono una legge generale di reciprocità”, “Si può assegnare un criterio generale per la determinazione dell’ordine delle varietà generate da forme proiettive;” “Il metodo di Jacobi per la soluzione dei problemi di Dinamica può applicarsi al problema della funicolare, quando le forze hanno una funzione potenziale.”
30. Lightly modified and published by Levi-Civita with the same title, “Sugli invarianti assoluti,” in *Atti Inst. Veneto*, 5 (1893–94), 1447–1523.
31. T. Levi-Civita, “Commemorazione,” 555.
32. ACS, MPI, DGIS, UIS, 1897–1910, busta 28.
33. Federico Enriques to Guido Castelnuovo, November 27, 1894, quoted in Pietro Nastasi and Rossana Tazzioli, eds. “Tullio Levi-Civita,” *Lettera matematica Pristem*, no. 57–58 (Milan, 2006).
34. Early in 1896, Levi-Civita had been appointed an internal professor in mathematics at the Scuola Normale in Scienze attached to the University of Pavia. He held that position until the end of the year.

35. Ernesto Pascal, Ferdinando Aschieri, Carlo Somigliana, “Relazione,” May 2, 1896, ACS, MPI, DGIS, 1897–1910, busta 28.
36. F. D’Arcais, G. Ricci, V. Volterra, “Relazione,” July 21, 1896, *ibid.* Translation by M. Vallisneri.
37. Eugenio Valli to unknown, November 22, 1896, ACS, MPI, DGIS, 1897–1910, busta 28.
38. Quoted in U. Bottazzini, A. Conte, and P. Gario, eds., *Riposte armonie* (Turin, 1996), footnote 348, which describes more fully this episode.
39. V. Volterra to Giovanni Battista Guccia, December 11, 1895, quoted in P. Nastasi, <http://www.treccani.it/enciclopedia/tullio-levi-civita> (2016).
40. Luca Gianturco to unknown, Jan. 7, 1898, Archivio Generale, Ateneo dell’Università degli Studi di Padova, fascicolo- docente Tullio Levi-Civita. Apparently, Marcolongo was a closet anti-Semite and nursed a long-standing grudge against Levi-Civita and other Jewish university professors in the mathematical community, which became more pronounced after the passage of Italy’s racial laws in 1938. For the details, see Giorgio Israel, *Il fascismo e la razza. La scienza italiana e le politiche razziali del regime* (Bologna, 2010), chapter 6.
41. V. Volterra, “Premio Lincei Matematica, 1909,” Vito Volterra Papers, Accademia Nazionale dei Lincei, Rome.
42. T. Levi-Civita, “Sulle trasformazioni delle equazioni dinamiche,” *Ann. di Mat.*, 24 (1896), 227.
43. U. Amaldi, “Tullio Levi-Civita,” *Opere matematiche*, vol. 1, xiii.
44. G. Ricci to T. Levi-Civita, July 27, 1918, Tullio Levi-Civita Papers, Accademia Nazionale dei Lincei, Rome.

Notes for Chapter 9

1. G. Ricci to B. Ricci, October 26, 1894, Fondo Ricci Curbastro, Lugo.
2. *Ibid.*, October 31, 1894.
3. *Ibid.*, November 3, 1894.
4. “It has been my experience that beneath the surface of every scientist there lurks a wounded person who never feels his work has been fully appreciated. Feynman was a rare, perhaps even unique, exception.” David Goodstein, “Richard Feynman: In Memoriam,” *California Tech*, February 19, 1988, p. 4.
5. T. Levi-Civita, “Commemorazione,” 558.
6. The prize was offered “to study the surfaces whose linear element can be reduced to the form: $ds^2 = [f(u)\phi(v)](du^2 + dv^2)$,” *Bull. Amer. Math. Soc.*, 2 (1893), 260–261.
7. E. Beltrami to G. Ricci, October 3, 1892, Fondo Ricci Curbastro, Lugo.

8. Gabriel Koenigs, *Notice sur les travaux scientifiques de Gabriel Koenigs* (Tours, 1897), 26. The following year, the French Academy gave him the Poncelet Prize for a group of works in geometry and mechanics.
9. G. Koenigs, “Résumé d’un mémoire sur les lignes géodésiques,” *Ann. Fac. Sc. de Toulouse* 6 (1892), 1-34.
10. G. Ricci, “Dei sistemi di coordinate atti a ridurre la espressione del quadrato dell’elemento lineare di una superficie alla forma $ds^2 = (U + V)(du^2 + dv^2)$,” *Rend. Acc. Lincei* 5 (1893), 73–81.
11. Jeremy Gray, “A History of Prizes in Mathematics,” in J. Carlson, A. Jaffe, and A. Wiles, eds., *The Millennium Prize Problems* (Providence, 2006), 16.
12. G. Ricci, “A proposito di una memoria sulle linee geodetiche del Sig. G. Koenigs,” *Rend. Acc. Lincei*, 5 (1893), 146–148.
13. G. Koenigs, “Réponse à la note de Monsieur le professeur Gregorio Ricci, du 3 septembre 1893,” *Rend. Acc. Lincei*, 5 (1893), 336.
14. G. Ricci, “Alcune parole a proposito della precedente risposta del sig. Koenigs,” *Rend. Acc. Lincei* 2 (1893), 353. An excerpt of Koenigs’s “Réponse à la note de M. le professeur Gregorio Ricci, du 3 septembre 1893,” *Rend. Acc. Lincei*, 2 (1893), 336–337, is reproduced as a footnote in Ricci, *Opere*, vol. 1, 351.
15. G. Koenigs, “Résumé,” *Ann. Fac. Sc. de Toulouse* 6(1892).
16. G. Ricci, “Sulla teoria delle linee geodetiche e dei sistemi isotermini di Liouville,” *Atti Ist. Veneto* 5 (1894), 356.
17. G. Koenigs, “Mémoire sur les lignes géodésiques,” *Mém. Savants Étrang.* 6 (1894), 1–318.
18. Richard Dedekind, a student of Gauss, was a German mathematician who made important contributions to the theory of the real number system in terms of arithmetic properties of the rational number system today known as “Dedekind cuts.”
19. Undated manuscript of T. Levi-Civita, III Raccoglitore, “Disputa Capelli-Ricci,” Fondo Ricci Curbastro, Lugo.
20. G. Ricci, “In memoria di Alfredo Capelli,” *Opere*, vol. 2, 331.
21. See Chapter 8, page 66 for a fuller discussion of this paper.
22. T. Levi-Civita to V. Volterra, April 8, 1896, in Pietro Nastasi and Rosanna Tazzioli, eds., *Aspetti scientifici e umani nella corrispondenza di Tullio Levi-Civita* (Milan, 2000), 22.
23. G. Ricci to T. Levi-Civita, July 3, 1896, Tullio Levi-Civita Papers, Accademia Nazionale dei Lincei, Rome.
24. G. Koenigs, “Mémoire sur les lignes géodésiques,” *Mém. Savants Étrang.*, No. 631 (1894), 318 pp.

25. G. Ricci to T. Levi-Civita, July 7, 1896, “La formazione,” in P. Nastasi and R. Tazzioli, eds., *Tullio Levi-Civita, Lettera matematica pristem* 57/58 (Milan, 2006), 13.
26. T. Levi-Civita to G. Ricci, July 9, 1896, *ibid.*
27. T. Levi-Civita, “Sulle trasformazioni,” 241.
28. T. Levi-Civita to G. Ricci, July 31, 1896, Fondo Ricci Curbastro, Lugo.
29. T. Levi-Civita, *Opere*, vol. 1, 5.
30. T. Levi-Civita to Arnold Sommerfeld, March 30, 1899, Sommerfeld Papers, Deutsches Museum (Archiv HS 1977-28/A, 200).
31. T. Levi-Civita to V. Volterra, November 28, 1899, in Nastasi and Tazzioli, “*Aspetti scientifici*,” 57.
32. G. Ricci and T. Levi-Civita, “Méthodes de calcul différentiel absolu et leurs applications,” *Math. Ann.* 54 (1900), 125–201.
33. “Ricci and Levi-Civita’s Tensor Analysis Paper,” in Robert Hermann, trans. and ed., *Lie Groups: History, Frontiers, and Applications*, vol. 2 (Brookline, 1975), iii. Hermann’s translation is a modern rendition of the original article, rather than a literal translation. The volume uses modern notation and terminology, starting with the substitution of the term “tensor analysis” for “absolute differential calculus.” The text also contains additional material, including a number of “Remarks” designed to bring the material into line with contemporary differential geometry and physics.
34. Quoted in G. Ricci and T. Levi-Civita, “Méthodes,” 128. Translation by D. Babbitt.
35. G. Ricci, *Lezioni sulla teoria delle superficie* (Padua, 1898); G. Ricci, *Lezioni sulla teoria matematica dell’elasticità*, completed in 1901 and published posthumously in 1957, in Ricci, *Opere*, vol. 2.
36. The Drucker brothers, Jewish booksellers who catered to the university community, ran their lively business from quarters in the historic palazzo del Bò.
37. George Oscar James, Review. *Bull. Amer. Math. Soc.* 7(1901), 359–360.
38. G. Ricci to V. Volterra, July 5, 1888, Vito Volterra Papers, Accademia dei Lincei.
39. G. Castelnuovo, “Commemorazione del socio Federigo Enriques,” *Rend. Acc. Naz. Lincei*, 8 (1947), 3–21. See also D. Babbitt and J. Goodstein, “Guido Castelnuovo and Francesco Severi: Two Personalities, Two Letters,” *Notices of the AMS*, 56 (2009), 800–801.
40. U. Bottazzini, “Ricci and Levi-Civita,” 248. See also, U. Bottazzini, A. Conte, and P. Gario, “La relazione di Castelnuovo ed Enriques. Documenti inediti per il premio Reale di Matematica del 1901,” *Studies in the History of Modern Mathematics*, III, *Suppl. Rendiconto del Circolo Matematico di Palermo*, 55 (1998), 75–156.

41. U. Bottazzini et al, “La relazione,” 127.
42. Ibid.
43. F. Enriques to G. Castelnuovo, April 18, 1903, in U. Bottazzini, A. Conte, and P. Gario, eds., *Riposte armonie. Lettere di Federigo Enriques a Guido Castelnuovo* (Turin, 1996), 559.
44. L. Bianchi, “Relazione sul concorso al Premio Reale per la Matematica per l’anno 1901,” *Atti Acc. Lincei Rend. Sedute solenni*, 2 (1902–1913), 142–151, on p. 147.
45. L. Bianchi, “Relazione,” 148.
46. Ibid., 149; G. Ricci, “Dei sistemi di congruenze ortogonali in una varietà qualunque,” *Opere*, vol. 2, 1–61.
47. L. Bianchi, “Relazione,” 150.
48. Ibid.

Notes for Chapter 10

1. Louis Kollros, “Albert Einstein en Suisse. Souvenirs,” *Helv. Phys. Acta Suppl.* 4 (1956), 278.
2. Abraham Pais, *Subtle Is the Lord: The Science and the Life of Albert Einstein* (New York, 1982), 201. See also the editorial note “Einstein on Gravitation and Relativity: The Static field,” *The Collected Papers of Albert Einstein* (CPAE hereafter), Vols. 1–14 (Princeton, 1987–).
3. CPAE, vol. 6, Doc. 42, p. 418.
4. In 1915, Einstein predicted twice that amount of bending, based on his newly formulated theory of general relativity.
5. Albert Einstein to Willem Julius, Sept. 22, 1911, in CPAE, vol. 5, Doc. 288, p.209.
6. In summer 1914, a solar-eclipse expedition led by the young German astronomer Erwin Finlay-Freundlich set out for the Crimea, only to be caught up in the outbreak of World War I before it could test Einstein’s prediction. Finlay-Freundlich was captured by Russian soldiers but released later in a prisoner exchange. The British full-solar-eclipse expeditions of 1919 provided an exceptional viewing opportunity since the eclipse positioned the sun in front of the Hyades, a particularly bright cluster of stars. For further detail, see Jeffrey Crelinsten, *Einstein’s Jury: The Race to Test Relativity* (Princeton, 2006).
7. See Stefanie U. Eminger, “Carl Friedrich Geiser and Ferdinand Rudio: The Men Behind the First International Congress of Mathematicians,” (Ph.D. diss., University of St. Andrew, 2015), 129, <http://hdl.handle.net/10023/6536>; A. Einstein, *Autobiographical Notes*, ed. and trans. Paul Arthur Schilpp (La Salle, 1979), 17; Anton Reiser, *Albert Einstein: A Biographical Portrait* (New York, 1930), 49.
8. Carl Seelig, *Albert Einstein* (London, 1956), 28. My thanks to Dennis Lehmkuhl for calling my attention to this quotation.

9. Peter L. Galison, "Minkowski's Space-Time: From Visual Thinking to the Absolute World," *Hist. Studies in the Phys. Sci.* 10 (1979), 96.
10. Hermann Minkowski, "Raum und Zeit," 1908, trans. D. Lehmkuhl, in *Minkowski Spacetime: A Hundred Years Later*, ed. Vesselin Petkov (Springer, 2010), xv–xlii, on xv.
11. Scott Walter, "Minkowski's Modern World," in V. Petkov, *Minkowski Spacetime*, 43–61, on 56; see also, S. Walter, "Minkowski, Mathematicians, and the Mathematical Theory of Relativity," in H. Goenner, J. Renn, J. Ritter, T. Sauer, eds. *The Expanding Worlds of General Relativity*, Einstein Studies, vol. 7 (1999), 45–86.
12. S. Walter, "Minkowski," 59.
13. Jean Eisenstaedt, *The Curious History of Relativity: How Einstein's Theory of Gravity Was Lost and Found Again* (Princeton, 2006), 44.
14. Anton Reiser, *Albert Einstein* (New York, 1930), 49.
15. A. Einstein, *Autobiographical Notes*, ed. and trans. Paul Arthur Schilpp (La Salle, 1996), 15.
16. Jakob Laub to A. Einstein, CPAE, vol. 5, Doc. 79, Feb. 2, 1908.
17. A. Pais, *Subtle Is the Lord*, 152. Pais is referring to the tensors of Gibbs and Heaviside, which are useful in physics, but have no relation to the curved manifolds addressed by the absolute differential calculus.
18. CPAE, vol. 2, Doc. 51, p. 329; see also the editorial note, "Einstein and Laub on the electrodynamics of moving media," vol. 2, pp. 503-507.
19. A. Einstein to Mileva Einstein-Marić, April 17, 1908, CPAE, vol. 5, Doc. 96, p. 69.
20. J. Laub to A. Einstein, May 18, 1908, CPAE, vol. 5, Doc. 101, pp. 72–73, on 73.
21. A. Einstein to J. Laub, July 30, 1908, CPAE, vol. 5, Doc. 113, pp. 81–82, on 82.
22. A. Einstein, "Fundamental Ideas and Methods of Relativity Theory, Presented in Their Development," [after 22 Jan. 1920], unpublished draft of an article for *Nature*, CPAE, vol. 7, Doc. 31.
23. A. Einstein, "How I Created the Theory of Relativity," trans. Y. A. Ono, *Physics Today*, 35(1982), 45–47, on 47. A new, definitive translation appears in CPAE, vol. 13, Doc. 399, see p. 638.
24. Philipp Frank, *Einstein: His Life and Times*, trans. George Rosen (New York, 1947), 82.
25. CPAE, vol. 4, Doc. 8, p. 133.
26. Quoted in Tilman Sauer, "Marcel Grossmann and his contribution to the general theory of relativity," p. 3, Apr. 22, 2014, arXiv:1312.4068v2 [physics. hist-ph].
27. *Ibid.*, 42.

28. More about Grossmann's early life is related in the recent biography by his granddaughter, Claudia E. Graf-Grossmann, *Aus Liebe zur Mathematik* (Zurich, 2015). An English-language edition is not currently available.
29. Banesh Hoffmann (with Helen Dukas), *Albert Einstein, Creator and Rebel*, (New York, 1972), 25.
30. Maja Winteler-Einstein, "Albert Einstein—A Biographical Sketch," CPAE, vol. 1, xxi.
31. Hermann Einstein to Jost Winteler, Dec. 30, 1895, CPAE, vol. 1, Doc. 14.
32. T. Sauer, "Marcel Grossmann," 4.
33. C. Seelig, *Albert Einstein*, 34.
34. *Ibid.*, 207-208.
35. P. Frank, *Einstein*, 102.
36. L. Kollros, "Albert Einstein," 278.
37. T. Sauer, "Marcel Grossmann," 42.
38. A. Pais, *Subtle Is the Lord*, 213.
39. *Ibid.*
40. T. Sauer, "Marcel Grossmann," 15.
41. Quoted in T. Sauer, *ibid.*, 16.
42. CPAE, vol. 6, Doc. 42, p. 418.
43. Quoted in S. U. Eminger, "Carl Friedrich Geiser," 21.
44. CPAE, vol. 13, Doc. 399, p. 638.
45. While he never got around to publishing these lectures, they are included in volume 2 of Ricci's *Opere*, published in 1957.
46. A. Einstein to A. Sommerfeld, October 29, 1912, quoted in J. Goodstein, "The Italian Mathematicians of Relativity," *Centaurus*, 26 (1983), 247.
47. Quoted in A. Pais, *Subtle Is the Lord*, 212.
48. A. Einstein and Marcel Grossmann, *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation* (Leipzig, 1913); CPAE, vol.4, Doc. 13.
49. Quoted in T. Sauer, "Marcel Grossmann," 44.
50. *Ibid.*, 18–19.
51. A. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie," *Annalen der Physik*, 49 (1916), 769-822 ["The Foundation of the General Theory of Relativity"].

52. T. Levi-Civita, “Gregorio Ricci-Curbastro,” *Opere*, vol. 1, 4.
53. *Ibid.*, 11.
54. J. Eisenstaedt, *The Curious History of Relativity*, 60.
55. These gravitational potentials are the components g_{ij} of the Riemannian metric tensor. In other words, the right-hand side represents non-gravitational physics; the left-hand side is the geometrical representation of gravity.
56. James Hartle, *Gravity: An Introduction to Einstein’s General Relativity* (Boston, 2003), 13.
57. Quoted in Walter Isaacson, *Einstein*, 196.

Notes for Chapter 11

1. T. Levi-Civita to Max Abraham, Feb. 7, 1907, in “1908 Congress (IV) of Mathematicians, Letters,” Levi-Civita Family Papers.
2. Max Born and Max von Laue, “Max Abraham,” *Physikalische Zeitschrift*, 24(1923), 53; see also Claudio Citrini, “Matematica e vita civile nel Politecnico di cento anni fa: la vicende di Max Abraham,” *Annali di Storia delle Università Italiane*, 12 (2008), 104.
3. The first volume (Leipzig, 1904–1905) was based on an earlier work by August Föppl on Maxwell’s theory of electricity (1894); later editions appeared in 1908, 1912, 1914, and 1918.
4. M. Abraham to T. Levi-Civita, Feb. 15, 1907, “1908 Congress,” Levi-Civita Family Papers.
5. For further detail, see Stanley Goldberg, *Understanding Relativity* (Boston, 1984), 103–149.
6. V. Volterra, “Le Matematiche in Italia nella seconda metà del secolo XIX,” in G. Castelnuovo, ed., *Atti del IV Congresso Internazionale dei Matematici*, vol. 1 (Rome, 1909), 64.
7. For further details of Abraham’s stay in America, see Lawrence Badash, ed., *Rutherford and Boltwood, Letters on Radioactivity* (New Haven, 1969), 193–194.
8. T. Levi-Civita to unknown recipient, draft, ca. 1909, Fondo Levi-Civita, Accademia Nazionale dei Lincei, Rome.
9. File, “Relazione della Commissione Giudicatrice, Concorso per Professore Straordinario di Meccanica Razionale nel R. Istituto Tecnico Superiore di Milano,” Oct. 8, 1909, Levi-Civita Family Papers.
10. A. Pais, *Subtle Is the Lord*, 156.
11. Quoted in A. Pais, *Subtle Is the Lord*, 232.
12. See Jagdish Mehra, “Einstein, Hilbert, and the Theory of Gravitation,” in J. Mehra, ed., *The Physicist’s Conception of Nature* (Dordrecht, 1973), 96–98.

13. A. Einstein to Lucien Chavan, ca. July 6, 1911, CPAE, vol. 5, Doc. 271, pp. 193–194.
14. A. Einstein to Heinrich Zangger, [before Feb. 29], 1912, CPAE, vol. 5, Doc. 366, p. 268.
15. A. Einstein to J. Laub, Aug. 11, 1911, CPAE, vol. 5, Doc. 275, p. 197.
16. For the best recent study of Abraham’s theory of gravitation, see Jürgen Renn, “The Summit Almost Scaled: Max Abraham as a Pioneer of a Relativistic Theory of Gravitation,” in J. Renn, ed., *The Genesis of General Relativity*, vol. 3 (Dordrecht, 2007), 1229–1255.
17. A. Einstein to Michele Besso, Mar. 26, 1912, CPAE, vol. 5, Doc. 377, pp. 276–279, on 278.
18. A. Einstein to H. Zangger, Jan. 27, 1915, vol. 5, Doc. 344, p. 250.
19. A. Einstein to Paul Ehrenfest, Feb. 12, 1912, vol. 5, Doc. 357, p. 260.
20. A. Einstein, “The Speed of Light and the Statics of the Gravitational Field,” *Annalen der Physik*, CPAE, vol. 4, Doc. 3, p. 95. In March, 1912, Einstein published his final paper on the static field, “On the Theory of the Static Gravitational Field,” CPAE, vol. 4, Doc. 4.
21. Carlo Cattani and Michelangelo De Maria, “Max Abraham and the Reception of Relativity in Italy: His 1912 and 1914 Controversies with Einstein,” in Don Howard and John Stachel, eds., *Einstein and the History of General Relativity* (Boston, 1896), 160–174, on 163.
22. A. Einstein, “Relativity and Gravitation: Reply to a Comment by M. Abraham,” *Annalen der Physik* 38(1912), 1059–1064; CPAE, vol. 4, Doc. 8. For a more detailed account, see the editorial note, “Einstein on Gravitation and Relativity: The Static Field,” vol. 4, pp. 122–128.
23. CPAE, vol. 4, Doc. 8, p. 133.
24. Quoted in C. Cattani and M. De Maria, “Abraham and Relativity,” 164.
25. A. Einstein, “Comment on Abraham’s Preceding Discussion ‘Once Again, Relativity and Gravitation,’ ” *Annalen der Physik* 39 (1912), 704; CPAE, vol. 4, Doc. 9.
26. T. Sauer to the author, Apr. 27, 2017.
27. Quoted in C. Cattani and M. De Maria, “Abraham and Relativity,” 165.
28. A. Einstein to M. Besso, Mar. 12, 1912, CPAE, vol. 5, Doc. 377.
29. A. Einstein to A. Sommerfeld, Oct. 29, 1912, vol. 5, Doc. 421.
30. A. Einstein to Alfred Kleiner, April 3, 1912, vol. 5, Doc. 382.
31. T. Levi-Civita to Giuseppe Colombo, Mar. 17, 1913, quoted in C. Citrini, “Matematica e Vita Civile,” 103.

32. *Ibid.*, 102.
33. Quoted in M. Born and M. von Laue, “Max Abraham,” *Phys. Zs.*, 24 (1923), 53.
34. A. Einstein to M. Besso, [after Jan. 1, 1914], Mar. 26, 1912, CPAE, vol. 5, Doc. 499.
35. Quoted in Barbara J. Reeves, “Einstein Politicized: The Early Reception of Relativity in Italy,” in Thomas F. Glick, ed., *The Comparative Reception of Relativity* (Dordrecht, 1987), 201.
36. F.G. Tricomi, “Matematici italiani del primo secolo dello stato unitario,” *Mem. Acc. Sci. Torino*, cl. sci., fis., 4th ser. 1(1962), 106.
37. C. Somigliana, *Tullio Levi-Civita e Vito Volterra* (Milan, 1946), 8.
38. A. Einstein, “The Formal Foundation of the General Theory of Relativity,” *Königlich Preussische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte*, CPAE, vol. 6, Doc. 9, p. 30. Unlike the title of the two previous *Entwurf* papers, Einstein replaced the word “generalized” with “general” in describing his theory of relativity.
39. Hanoch Gutfreund and Jürgen Renn, *The Road to Relativity: The History and Meaning of Einstein’s “The Foundation of General Relativity,”* (Princeton, 2015), 26.
40. A. Einstein, “The Formal Foundation,” CPAE, vol. 6, Doc. 9.
41. M. Abraham to T. Levi-Civita, Feb. 23, 1915, Tullio Levi-Civita Papers, Lincei.
42. A. Pais, *Subtle Is the Lord*, 244.
43. C. Somigliana, “Tullio Levi-Civita,” 6–7.
44. For extensive analysis, see C. Cattani and M. De Maria, “The 1915 Epistolary Controversy between Einstein and Tullio Levi-Civita,” in Howard and Stachel, eds., *Einstein and the History of General Relativity*, vol. 1, 175–200, on 194.
45. John Norton, “How Einstein Found His Field Equations, 1912–1915,” *ibid.*, 101–159, on 138.
46. H. Lorentz, Jan. 1, 1916, CPAE, vol. 8, Doc. 177.
47. A. Einstein to T. Levi-Civita, Mar. 5, 1915, CPAE, vol. 8, Doc. 60, p. 71. Portions of the material on Einstein and Levi-Civita previously appeared in the journal *Centaurus* and are reprinted here in slightly different form with the consent of John Wiley & Sons.
48. T. Sauer, “Albert Einstein, Review Paper on General Relativity Theory (1916),” I. Grattan-Guinness, ed., *Landmark Writings in Western Mathematics, 1640–1940*, chap. 63 (Amsterdam, 2005), 811.
49. A. Einstein to T. Levi-Civita, Mar. 17, 1915, CPAE, vol., 8, Doc. 62, p. 73.
50. A. Einstein to T. Levi-Civita, Mar. 26, 1915, CPAE, vol. 8, Doc. 66.

51. M. Abraham to T. Levi-Civita, Mar. 30, 1915, Tullio Levi-Civita Papers, Lincei.
52. A. Einstein to T. Levi-Civita, Apr. 2, 1915, CPAE, vol. 8, Doc. 69.
53. A. Einstein to H. Zangger, April 10, 1915, CPAE, vol. 8, Doc. 73.
54. A. Einstein to T. Levi-Civita, Apr. 14, 1915, CPAE, vol 8, Doc., 75.
55. For further details, see C. Citrini, “Matematica e Vita Civile,” 109–115.
56. M. Abraham to T. Levi-Civita, Apr. 27, 1915, Tullio Levi-Civita Papers, Lincei.
57. *Ibid.*, Oct. 16, 1919.
58. See M. Abraham’s letter to Theodore von Kármán, Feb. 16, 1922, Theodore von Kármán Papers, Institute Archives, California Institute of Technology, Box 1.1.
59. T. von Kármán to A. Einstein, Feb. 22, 1922, CPAE, vol. 13, Doc. 61 and “Auszüge Aus Den Gutachten Über Max Abraham,” undated, but after Feb. 22, in T. von Kármán Papers, Abraham file, Box 1.1.
60. W. Isaacson, *Einstein*, 214.
61. A. Einstein to A. Sommerfeld, Nov. 28, 1915, CPAE, vol. 8, Doc. 153.
62. A. Einstein, “On the General Theory of Relativity,” Nov. 4, 1915, CPAE, vol. 6, Doc. 21.
63. A. Einstein to P. Ehrenfest, Jan. 17, 1916, CPAE, vol. 8, Doc. 182.
64. A. Pais, *Subtle Is the Lord*, 253.
65. *Ibid.*
66. J. Norton, “How Einstein Found His Field Equations,” 148.
67. *Ibid.*, 142.
68. A. Einstein to A. Sommerfeld, Nov. 28, 1915, CPAE, vol. 8, Doc. 153.
69. A. Einstein, “On the General Theory,” Nov. 4, 1915, CPAE, vol. 6, Doc. 21.
70. A. Einstein, “The Field Equations of Gravitation,” Nov. 25, 1915, CPAE, vol. 6, Doc. 25.
71. A. Einstein, “The Foundation of the General Theory of Relativity,” Mar. 20, 1916, CPAE, vol. 6, Doc. 30.
72. *Ibid.*, p. 153.

Notes for Chapter 12

1. Libera and Tullio Levi-Civita to Trevisani family, August 2, 1914, Levi-Civita family correspondence.
2. Claudio G. Segrè, Italo Balbo: *A Fascist Life* (Berkeley, 1987), 18.

3. R. J. B. Bosworth, *Mussolini's Italy: Life under the Fascist Dictatorship, 1915–1945* (New York, 2007), 58.
4. S.S.C. Interview.
5. Ibid.
6. Ibid.
7. Ibid.
8. Ibid.
9. Ibid.
10. See for example, U. Bottazzini, “Ricci and Levi-Civita,” 254.
11. T. Levi-Civita to George Birkhoff, Jan. 9, 1917, quoted in J. Goodstein, “The Italian Mathematicians of Relativity,” 255.
12. T. Levi-Civita, “Nozione di parallelismo in una varietà qualunque e conseguente specificazione geometrica della curvatura Riemanniana,” *Rend. Circ. Mat. di Palermo* 42 (1917), 173–215; T. Levi-Civita, *Opere matematiche*, vol. 4 (Rome, 1960), 1–39.
13. Ibid., 197.
14. Ibid.
15. G. Castelnuovo to T. Levi-Civita, April 19, 1917, Tullio Levi-Civita Papers, Accademia Nazionale dei Lincei, Rome. For Severi’s approach to the notion of parallelism, see F. Severi, “Sulla curvatura delle superficie e varietà,” *Rend. Circolo Mat. Palermo* 42 (1917), 227–259.
16. U. Bottazzini, “Ricci and Levi-Civita,” 254.
17. O. Darrigol, “The Mystery of Riemann’s Curvature,” 77.
18. George Y. Rainich, “Levi-Civita on Tensor Calculus,” *Bull. Amer. Math. Soc.* 34 (1928), 775–776. For more on this fundamental contribution of Levi-Civita to the advance in the absolute differential calculus, see W. V. D. Hodge, “Tullio Levi-Civita, 1873–1941,” Appendix C.
19. In simpler, modern terms, one speaks of a curved space embedded in a larger-dimensional Euclidian (i.e., flat) space. To move vectors along the curved surface, one follows their motion in the embedding space while projecting them “down” to the surface. Nevertheless, a manifold’s Riemann curvature prescribes the displacement of vectors even when the manifold is not embeddable.
20. Harold S. Ruse, “Tullio Levi-Civita,” *Edinburgh Math. Notes* 33 (1943), 21.
21. D. Struik, “Schouten, Levi-Civita, and the Emergence of Tensor Calculus,” in David Rowe and John McCleary, eds., *The History of Modern Mathematics*, vol. 2 (Boston, 1989), 103.
22. Ibid., 103–104.

23. Jan A. Schouten, *Der Ricci-Kalkül* (Berlin, 1924); J.A. Schouten, *Ricci-Calculus: An Introduction to Tensor Analysis and its Geometrical Applications* (Berlin, 1954).
24. Enrico Sangiorgi to the author, Mar. 24, 2014; Armanda Capucci, “La vecchia casa di Gregorio Ricci Curbastro,” [Internet], retrieved June 23, 2014, available from <http://www.pavaglione.lugo.net/2014/06/la-vecchia-casa-di-gregorio-ricci.html>.
25. G. Ricci to Giorgio Ricci Curbastro, Nov. 26, 1915, Fondo Ricci Curbastro, Lugo.
26. G. Ricci to Cesare Ricci Curbastro, Feb. 24, 1916, Fondo Ricci Curbastro, Lugo.
27. Cesare later met and married Giulia Archi.
28. G. Ricci to Giorgio Ricci Curbastro, September 10, 1917, Fondo Ricci Curbastro, Lugo.
29. S.S.C. Interview.
30. G. Castelnuovo to T. Levi-Civita, Aug. 1, 1918, in the Levi-Civita Papers, Rome; ACS, MPI, DGIU, fasc. professori universitaria, III serie, busta 267.
31. T. Levi-Civita, “Come potrebbe un conservatore giungere alla soglia della nuova meccanica,” *Opere matematiche*, vol. 4, 197.
32. *Il Messaggero*, Oct. 30, 1921.
33. G. Ricci to T. Levi-Civita, Oct. 31, 1921, Levi-Civita Papers, Rome.
34. “Il Prof. Einstein a Padova,” *Corriere della Sera*, October 28, 1921.
35. F. Toscano, *Il genio*, 253–255.
36. F. Tricomi, *La mia vita di matematico attraverso la cronistoria dei miei lavori* (Padova, 1967), 7.
37. Fragment of Ricci’s inscription on Elio’s gravestone, quoted in Armanda Capucci, “La vecchia casa di Gregorio Ricci Curbastro,” <http://www.pavaglione.lugo.net/2014/06/la-vecchia-casa-di-gregorio-ricci.html> [retrieved September 29, 2014].
38. T. Levi-Civita, “Commemorazione,” 562.
39. G. Ricci, “Direzioni e invarianti principali in una varietà qualunque,” *Opere*, vol. 2, 315.
40. Marcel Berger, *A Panoramic View of Riemannian Geometry* (Berlin, 2003), 245.
41. Applications of Ricci’s curvatures in Riemannian geometry mostly stayed under the mathematical world’s radar until 1940, when S. B. Myers, “Riemannian Manifolds with Positive Mean Curvature,” *Duke Math. Journal* 8(1941), 401–404, obtained a significant new result in Riemannian geometry using Ricci’s curvature tensor.
42. T. Levi-Civita, “Sulla espressione analitica spettante al tensore gravitazionale nella teoria di Einstein,” in *Opere matematiche*, vol. 4, 63.
43. T. Levi-Civita, *The Absolute Differential Calculus*, ed. Enrico Persico and transl. Marjorie Long (New York: Dover Publications, 1977), 200.

44. Enrico Bompiani, “La géométrie des espaces courbes et le tenseur d’énergie d’Einstein,” *Comptes rendus* 174 (1922), 737. I thank Tilman Sauer for this reference.
45. Quoted in Solomon Lefschetz, “Luther Pfahler Eisenhart, 1876–1965,” *Biographical Memoirs of the National Academy of Sciences* 40 (1969), 71.
46. H. S. Ruse, “The Ricci Calculus,” *Nature* 171 (1953), 62.
47. Luther P. Eisenhart to Oswald Veblen, May 7, 1912, Oswald Veblen Papers, Manuscript Division, Library of Congress, Washington, D.C.
48. Leopold Infeld, *Quest: The Evolution of a Scientist* (London, 1941), 238.
49. L. P. Eisenhart, “Ricci’s Principal Directions for a Riemann Space and the Einstein Theory,” *Proc. Nat. Acad. Sci.* 8 (1922), 19.
50. L. P. Eisenhart, *Riemannian Geometry* (Princeton, 1926), iii.
51. D. Struik, “J.A. Schouten and the tensor calculus,” *Nieuw Archief Voor Wiskunde* 26 (1978), 103.
52. G. Ricci, “Sulle varietà a invarianti principali equali,” *Rend. Acc. Lincei* 33 (1924), 431.
53. J. Eisenstaedt, *The Curious History of Relativity*, 131.
54. The Einstein equations (or equivalently, the action, written in terms of the Ricci scalar, the contraction of the Ricci tensor) are also subsumed in string theory, quantum gravity, and alternative theories of gravity.
55. Dana Mackenzie, “The Poincaré Conjecture—Proved,” *Science* 314(2006), 1848–1849.
56. G. Ricci, “Sulle varietà,” 432.
57. For an introduction to the historical issues, see David Rowe, “Einstein’s Gravitational Field equations and the Bianchi Identities,” *Mathematical Intelligencer* 24(2002), 57–66.
58. F. Toscano, “Luigi Bianchi, Gregorio Ricci Curbastro e la scoperta delle identità di Bianchi,” *Atti del XX congresso nazionale di storia della fisica e dell’astronomia*, CUEN, Napoli (2001), 361.
59. T. Levi-Civita, *The Absolute Differential Calculus*, ed. Enrico Persico (New York, Dover Publications, 1977), 182.
60. T. Levi-Civita, “Commemorazione,” 564.
61. H. S. Ruse, “The Ricci Calculus,” *Nature* 171 (1953), 61–62.
62. F. G. Tricomi, Remarks, in “Tullio Levi-Civita, Convegno internazionale celebrativo del centenario della nascita,” *Atti dei Convegni Lincei* 8 (1975), 18–19.
63. Benito Mussolini, quoted in R. J. Bosworth, *Mussolini’s Italy*, 240.

64. For an account of Volterra's lonely defiance of Mussolini's regime, see J. Goodstein, *The Volterra Chronicles: The Life and Times of an Extraordinary Mathematician* (Providence, 2007), 193–200.
65. Giuseppe Levi to T. Levi-Civita, November 8, 1931, quoted in P. Nastasi, "La comunità matematica italiana di fronte alle legge razziali," in Massimo Galuzzi, ed., *Giornate di storia della matematica: Cetraro, 1988* (Rende, 1991), 438.
66. T. Levi-Civita to Pietro de Francisci, November 19, 1931, "Giuramento, Archivio," Levi-Civita family papers.
67. W. V. D. Hodge, "Tullio Levi-Civita." See Appendix C.
68. J. Goodstein and D. Babbitt, "A Fresh Look at Francesco Severi," *Notices Math. Soc.* 59 (2012), 1070. Neugebauer subsequently resigned from all editorial duties at Springer journals, left Germany, and joined the Brown university faculty in 1939, where he became the founding editor of the AMS's new abstracting journal, *MR* (*Mathematical Reviews*).
69. S.S.C. Interview.
70. C. Somigliana, "Tullio Levi-Civita e Vito Volterra," *Rend. Seminario matematico e fisico di Milano* 17 (1946), 2.
71. D. Struik to the author, November 6, 1973.

Notes for Appendix A

1. E. P. Wigner. The unreasonable effectiveness of mathematics in the natural sciences. *Communications on Pure and Applied Mathematics*, 13(1): 1–14, 1960.
2. A. Einstein and M. Grossmann. *Outline of a generalized theory of relativity and of a theory of gravitation*. Teubner, Leipzig, 1913.
3. Einstein on gravitation and relativity: the collaboration with Marcel Grossmann. In M. J. Klein, A. J. Kox, J. Renn, and R. Schulman, editors, *The Collected Papers of Albert Einstein*, volume 4, page 294. Princeton University Press, Princeton, NJ, 1995.
4. A. Einstein. The field equations of gravitation. *Sitzungsberichte Königlich Preussische Akademie der Wissenschaften (Berlin)*, page 844, 1915.
5. K. Reich. Differential geometry. In I. Grattan-Guinness, editor, *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, volume 2, page 331. Johns Hopkins, Baltimore and London, 2003.
6. A. Zee. *Einstein gravity in a nutshell*. Princeton University Press, Princeton, NJ, 2013.
7. R. Cooke. *It's about time: elementary mathematical aspects of relativity*. American Mathematical Society, Providence, RI, 2017.
8. J. Renn, editor. *The Genesis of General Relativity*. Springer, Dordrecht, The Netherlands, 2007.

9. Einstein's research notes on a generalized theory of relativity. In M. J. Klein, A. J. Kox, J. Renn, and R. Schulman, editors, *The Collected Papers of Albert Einstein*, volume 4, page 192. Princeton University Press, Princeton, NJ, 1995.
10. G. Ricci and T. Levi-Civita. Méthodes de calcul différentiel absolu et leurs applications. *Mathematische Annalen*, 54:125–201, 1900.
11. A. Einstein. On the general theory of relativity. *Sitzungsberichte Königlich Preussische Akademie der Wissenschaften (Berlin)*, page 778, 1915; A. Einstein. Explanation of the perihelion motion of mercury from the general theory of relativity. *Sitzungsberichte Königlich Preussische Akademie der Wissenschaften (Berlin)*, page 831, 1915; A. Einstein. The field equations of gravitation. *Sitzungsberichte Königlich Preussische Akademie der Wissenschaften (Berlin)*, page 844, 1915.
12. One equation that uses the scalar product is the law of motion for a particle in a central potential,

$$m \frac{d^2 x^i}{dt^2} = -\frac{\partial}{\partial x^i} U(r),$$

with $r^2 = \sum_i (x^i)^2$.

13. These forces are termed *fictitious* because they do not arise from physical interactions between bodies, but rather from describing physics from a non-inertial coordinate frame—in this case a rotating frame. Since these forces originate from geometry (more precisely, from kinematics), they affect bodies of different mass in the same way, as can be seen by dividing both sides of Eq. (7) by m .
14. A. Einstein. On the electrodynamics of moving bodies. *Annalen der Physik*, 17:891, 1905.
15. H. Minkowski. Space and time. *Physikalische Zeitschrift*, 10:75, 1908.
16. By contrast, in pre-relativistic physics Δt^2 and $\Delta x^2 + \Delta y^2 + \Delta z^2$ would have been separately invariant with respect to rotations and translations.
17. H. Minkowski. Space and time. *Physikalische Zeitschrift*, 10:75, 1908.
18. G. Ricci and T. Levi-Civita. Méthodes de calcul différentiel absolu et leurs applications. *Mathematische Annalen*, 54:125–201, 1900.
19. R. Hermann. *Ricci and Levi-Civita's tensor analysis paper: translation, comments, and additional material*. Math Sci Press, Brookline, MA, 1975.
20. These are the distances that would be measured by local observers with their rulers, or in the case of relativistic space-time, with their clocks and rulers. In the *Entwurf*, these distances are called “naturally measured.”
21. In denoting the transformation, we omit indices to indicate that *all* the y^i (for $i = 1, \dots, N$) are functions of *all* the x^i .
22. In writing these formulas we work at the “same” manifold point, identified by the coordinates x_0 or transformed coordinates $y_0 = y(x_0)$.

23. E. B. Christoffel. *Über die Transformation der homogenen Differentialausdrücke zweiten Grades*, *Journal für die reine und angewandte Mathematik*, 70:46–70, 1869.
24. The Christoffel symbol is neither covariant or contravariant; rather, it “completes” a partial derivative with the exact term needed to make it a covariant object. The symbol has an intuitive geometrical interpretation for embedded surfaces, it projects the partial derivative, which may include a normal component, back into the surface. However RLC and their contemporaries had an algebraic rather than geometric understanding of covariant differentiation. Levi-Civita himself would eventually make the connection to geometry in 1917.
25. Incidentally, the metric and inverse metric can be used to lower and raise indices: $A_i = g_{ij}A^j$ is covariant, while $f^i = g^{ij}f_j$ is contravariant.
26. G. Ricci and T. Levi-Civita. *Méthodes de calcul différentiel absolu et leurs applications*. *Mathematische Annalen*, 54:125–201, 1900.
27. Another result with a beautiful geometrical interpretation that was not understood by RLC in 1900. See, e.g., A. Zee. *Einstein gravity in a nutshell*. Princeton University Press, Princeton, NJ, 2013.
28. More correctly, *pseudo-Riemannian* manifold, since the metric of space-time allows for negative proper distances along “time-like” directions. This is a distinction ripe with physical consequences, but one that can largely be ignored for the purpose of mathematical development.
29. Local Lorentz coordinates make direct physical sense with respect to the measurements of rods and clocks. Beyond local neighborhoods, however, coordinates lose physical meaning. Einstein initially fights, then comes to terms with this proposition—a fascinating story that is beyond our scope here. See, e.g., J. Norton, Einstein’s triumph over the space-time coordinate system. *Diálogos*, 79:253, 2002.
30. I.e., point masses so light that they do not themselves affect the gravitational field.
31. A. Einstein and M. Grossmann. *Outline of a generalized theory of relativity and of a theory of gravitation*. Teubner, Leipzig, 1913.
32. The ten unique components of the metric (a symmetric 4-by-4 tensor) provide a redundant description of the gravitational degrees of freedom, and this redundancy must be removed before one can make physical predictions. One way to remove the redundancy is to impose additional conditions on one’s choice of coordinates.
33. J. D. Norton. What was Einstein’s “fateful prejudice”? In J. Renn, editor, *The Genesis of General Relativity*, page 715. Springer Netherlands, Dordrecht, 2007.
34. A. Einstein and M. Grossmann. *Outline of a generalized theory of relativity and of a theory of gravitation*. Teubner, Leipzig, 1913.
35. M. Janssen and J. Renn. Untying the knot: how Einstein found his way back to field equations discarded in the Zurich notebook. In J. Renn, editor, *The Genesis of General Relativity*, page 839. Springer Netherlands, Dordrecht, 2007.

36. G. Ricci and T. Levi-Civita. Méthodes de calcul différentiel absolu et leurs applications. *Mathematische Annalen*, 54:125–201, 1900.
37. See, for instance, C. W. Misner, K. S. Thorne, and J. A. Wheeler. *Gravitation*. W. H. Freeman, San Francisco, 1973, where this *modus operandi* surfaces continually.
38. A. Einstein. *Autobiographical notes*. Open Court, La Salle, IL, 1949.
39. M. Janssen, J. Renn, T. Sauer, J. D. Norton, and J. Stachel. A commentary on the notes on gravity in the Zurich notebook. In J. Renn, editor, *The Genesis of General Relativity*, page 489. Springer Netherlands, Dordrecht, 2007.
40. In the *Entwurf*, Grossmann goes as far as stating that “I have purposely not employed geometrical aids because, in my opinion, they contribute very little to an intuitive understanding of the conceptions of vector analysis.”
41. G. Hessenberg. Vektorielle Begründung der Differentialgeometrie. *Mathematische Annalen*, 78:187, 1917.
42. T. Levi-Civita. Nozione di parallelismo in una varietà qualunque. *Rend. del Circ. Mat. di Palermo*, 17:173, 1917.
43. H. Weyl. *Raum. Zeit. Materie. Vorlesungen über allgemeine Relativitätstheorie*. Springer, Berlin, 1918.
44. See, e.g., A. Zee. *Einstein gravity in a nutshell*. Princeton University Press, Princeton, NJ, 2013.
45. K. Reich. Differential geometry. In I. Grattan-Guinness, editor, *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, volume 2, page 331. Johns Hopkins, Baltimore and London, 2003.
46. A point made, e.g., by F. Toscano, *Il genio e il gentiluomo: Einstein e il matematico italiano che salvò la teoria della relatività generale*, Alpha Test, Milano, 2004.

Notes for Appendix B

1. It was not published. The manuscript, found among Ricci’s papers, was kindly placed at my disposal by the family. [Unless otherwise indicated, all footnotes are from Levi-Civita’s original memoir. The manuscript to which he alludes is now held by the Liceo scientifico statale “G. Ricci Curbastro,” Lugo (RA).]
2. One need only recall the book by J. A. Schouten entitled *Der Ricci-Kalkul* (Berlin: Springer, 1924).
3. *Translators’ Note*: Now called tensors.
4. E. Beltrami, “Relazione sul concorso al Premio Reale per la Matematica per l’anno 1887,” *Rendiconti dell’ Accademia Lincei* 5 (1889), 300–308, on pp. 304–307.
5. L. Bianchi, “Relazione sul concorso al Premio Reale, del 1901, per la Matematica,” *Atti dell’Accademia dei Lincei. Rendiconti dell’adunanza solenne del 1 giugno 1902*, 2, 142–151, on pp. 147–150.

6. G. Ricci Curbastro, *Sulle condizioni idrauliche della campagna a destra del Reno-primario e sui provvedimenti atti a migliorarle*, Faenza, Conti, 1881, pp. 1–36.
7. G. Ricci Curbastro, *Nuova Relazione al Consiglio Comunale sulla Proposta di condurre a Lugo le acque delle Vallette*, Lugo, Cremonini, 1903, pp. 1–41.
8. *Translators' Note*: There is no record of this usage in English; in Italian it meant the parts of algebra that were most useful for analysis.
9. G. Ricci-Curbastro, *Lezioni di analisi infinitesimal: funzioni di una variabile*, Padua, 1926.
10. “Sul concetto di successione in relazione col teorema fondamentale del calcolo integrale,” *Atti. R. Ist. Veneto*, 69(1910), p. 1055.
Translators' Note: It is not altogether clear why Levi-Civita singles out this exposition (published in Ch. 2 of the 1926 textbook) as especially original, given that Ricci follows quite closely the standard definition of the Darboux integral. The derivation does however embody the principle, dear to Ricci, of the “unity of the calculus around the fundamental concept of sequence” (Ricci 1926, in the unfinished preface); this may be the reason why Levi-Civita would consider it especially characteristic of Ricci.
11. *Translators' Note*: We believe this is referring to the famous Ricci-TLC paper published in 1900.
12. *Translators' Note*: The two contracted Riemann tensors referred to here are the contraction R_{ij} of the Riemann tensor $R_{ij}{}^{kl}$ for any Riemannian manifold by Ricci (1904), while the second one is the contraction of the Riemann tensor for any 4 dimensional *semi-Riemannian* manifold of signature (1,3) (i.e. space-time) by Einstein and Grossman (1913). Since Ricci’s tensor was defined nine years earlier than Einstein and Grossman’s, this tensor is now almost always referred to as the Ricci tensor (or curvature) for any semi-Riemannian manifold.
13. *Translators' Note*: The *Einstein tensor* is defined by $G_{ij} := R_{ij} - kg_{ij}R$ where R is the *scalar Ricci tensor*, g_{ij} is the Lorentzian pseudo-metric on space time, and k is a suitable constant.
14. *Translators' Note*: In the usual inertial coordinates, $ds^2 = dx^2 + dy^2 + dz^2 - dt^2$.
15. Friedrich Kottler, “Über die Raumzeitlinien der Minkowski’schen Welt,” *Kaiserliche Akademie der Wissenschaften (Vienna), Mathematisch-naturwissenschaftliche Klasse, Abteilung IIa. Sitzungsberichte* 121 (1912): 1659–1759.
16. *Translators' Note*: Usually written as $ds^2 = \sum_{i,j=1,2,3,4} g_{ij} dx^i dx^j$, where the $\{g_{ij}\}$ are the unknowns.
17. Albert Einstein and Marcel Grossmann, *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation*, Leipzig, 1913.

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Courtesy of Bill Gross.



In the first decade of the twentieth century as Albert Einstein began formulating a revolutionary theory of gravity, the Italian mathematician Gregorio Ricci was entering the later stages of what appeared to be a productive if not particularly memorable career, devoted largely to what his colleagues regarded as the dogged development of a mathematical language he called the absolute differential calculus. In 1912, the work of these two dedicated scientists would intersect—and physics and mathematics would never be the same. *Einstein's Italian Mathematicians* chronicles the lives and intellectual contributions of Ricci and his brilliant student Tullio Levi-Civita, including letters, interviews, memoranda, and other personal and professional papers, to tell the remarkable, little-known story of how two Italian academicians, of widely divergent backgrounds and temperaments, came to provide the indispensable mathematical foundation—today known as the tensor calculus—for general relativity.

A wonderfully written chronicle of the lives of two great mathematicians and how their work shaped Einstein's masterpiece as well as ushering in new fields of mathematics. The book is also an intriguing and insightful portrait of Italy during the period from Italian independence in 1870 until the onset of World War II.

—Gino Segre, *Physics Department, University of Pennsylvania*

Galileo said that mathematics is the language of nature. Einstein might have found himself mute when it came to describing gravity if it weren't for the mathematics of covariant derivatives developed by Galileo's countrymen Gregorio Ricci-Curbastro and Tullio Levi-Civita. Judy Goodstein tells their stories and their connection to Einstein with clarity and grace in a most readable book.

—Barry Simon, *California Institute of Technology*

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